The importance of friction in mountain wave drag enhancement by parametric resonance

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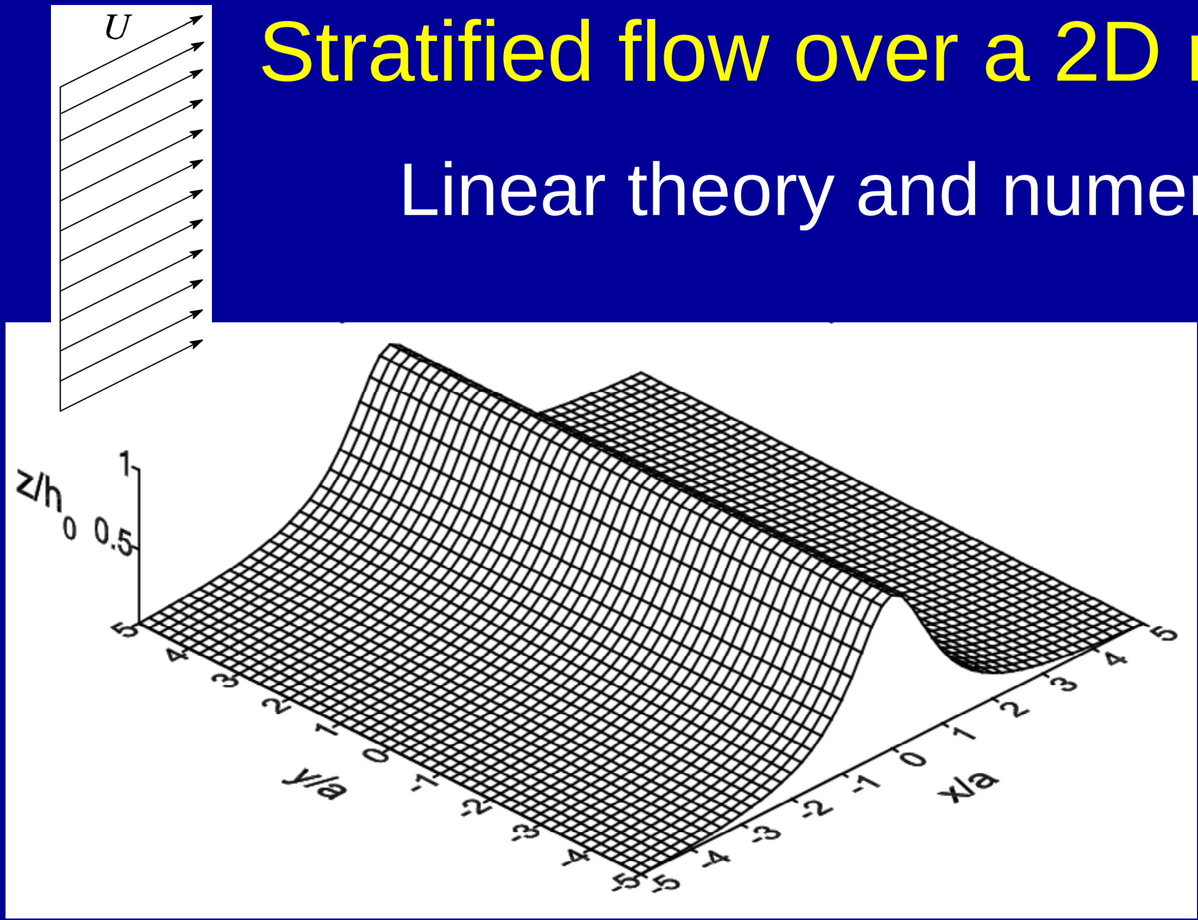
Stratified flow over a 2D mountain

Linear theory and numerical simulations



Bell-shaped ridge

$$h = \frac{h_0}{1 + (x/a)^2}$$

Basic Scorer parameter: $l_0 = N/U$

$$N^2 = \frac{g}{\theta_0}\frac{d\overline{\theta}}{dz}$$

Linear model

- Boussinesq approximation
- Linearization ($\boldsymbol{l_0 h_0 << 1}$)
- Non-hydrostatic flow ($\boldsymbol{l_0 a \neq \infty}$)
- Viscous processes represented by Rayleigh damping λ

Linear model

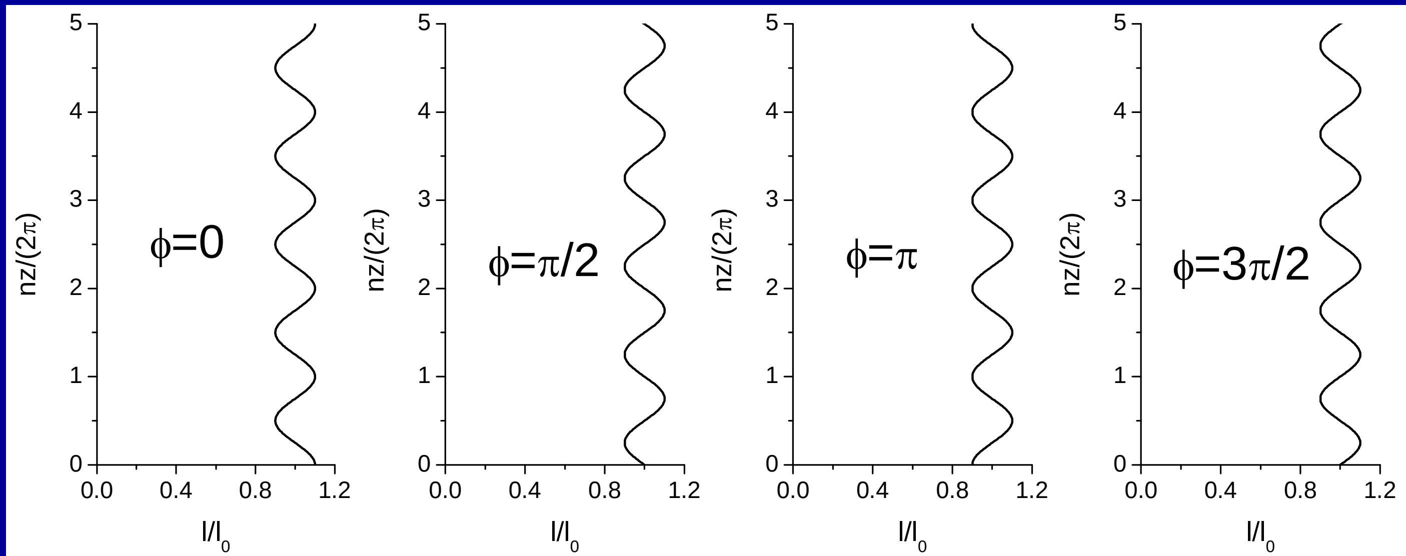
$$\hat{w}'' + \left(\frac{l^2}{1 - i\lambda / Uk} - k^2 \right) \hat{w} = 0$$

Taylor-Goldstein equation

- Boundary condition at surface: $\hat{w}(z=0) = iUk\hat{h}$
- Radiation or decay boundary condition aloft

Used Scorer parameter profiles: $l^2 = l_0^2 \left[1 + \varepsilon \cos(nz + \phi) \right]$

$\varepsilon = 0.1$



Calculation of gravity wave drag:

$$D = \int_{-\infty}^{+\infty} p(z=0) \frac{\partial h}{\partial x} dx = 2\pi i \int_{-\infty}^{+\infty} k \hat{p}^* (z=0) \hat{h} dk$$

Solution procedure

Expand solution in power series of ε :

$$\hat{w} = \hat{w}_0 + \varepsilon \hat{w}_1 + \varepsilon^2 \hat{w}_2 + \dots$$

Drag is also power series of ε :

$$D = D_0 + \varepsilon D_1 + \varepsilon^2 D_2 + \dots$$

It is sufficient to expand D up to first order:

$$\frac{D}{D_0} = 1 + \varepsilon \frac{D_1}{D_0} = 1 + 2\varepsilon \frac{\int_0^{+\infty} k' e^{-2k'} m'_R \frac{(4m'^2_R + 4m'^2_I - n'^2)\cos\phi - 4n'm'_I \sin\phi}{(4m'^2_R - 4m'^2_I - n'^2)^2 + 64 m'^2_R m'^2_I} dk'}{\int_0^{+\infty} k' e^{-2k'} (m'_R + \lambda a/(Uk') m'_I) dk'}$$

$k' = ka$ $\quad$ $n' = n/l_0$ $\quad$ $m'_R = m_R / l_0$ $\quad$ $m'_I = m_I / l_0$

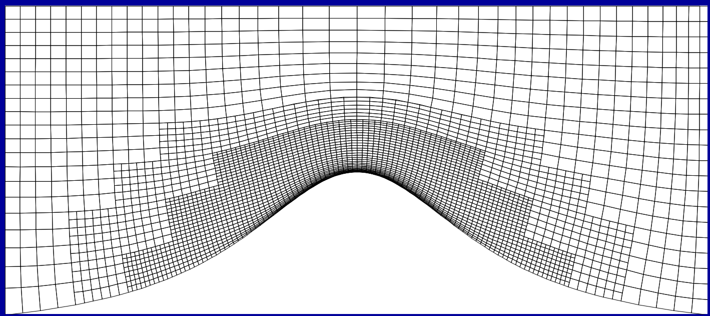
$m = m_R + i m_I$ Vertical wavenumber of internal gravity waves

Normalized drag D/D_0 is function of: ε, n/l_0, ϕ, $l_0 a$, $\lambda a/U$

Numerical simulations

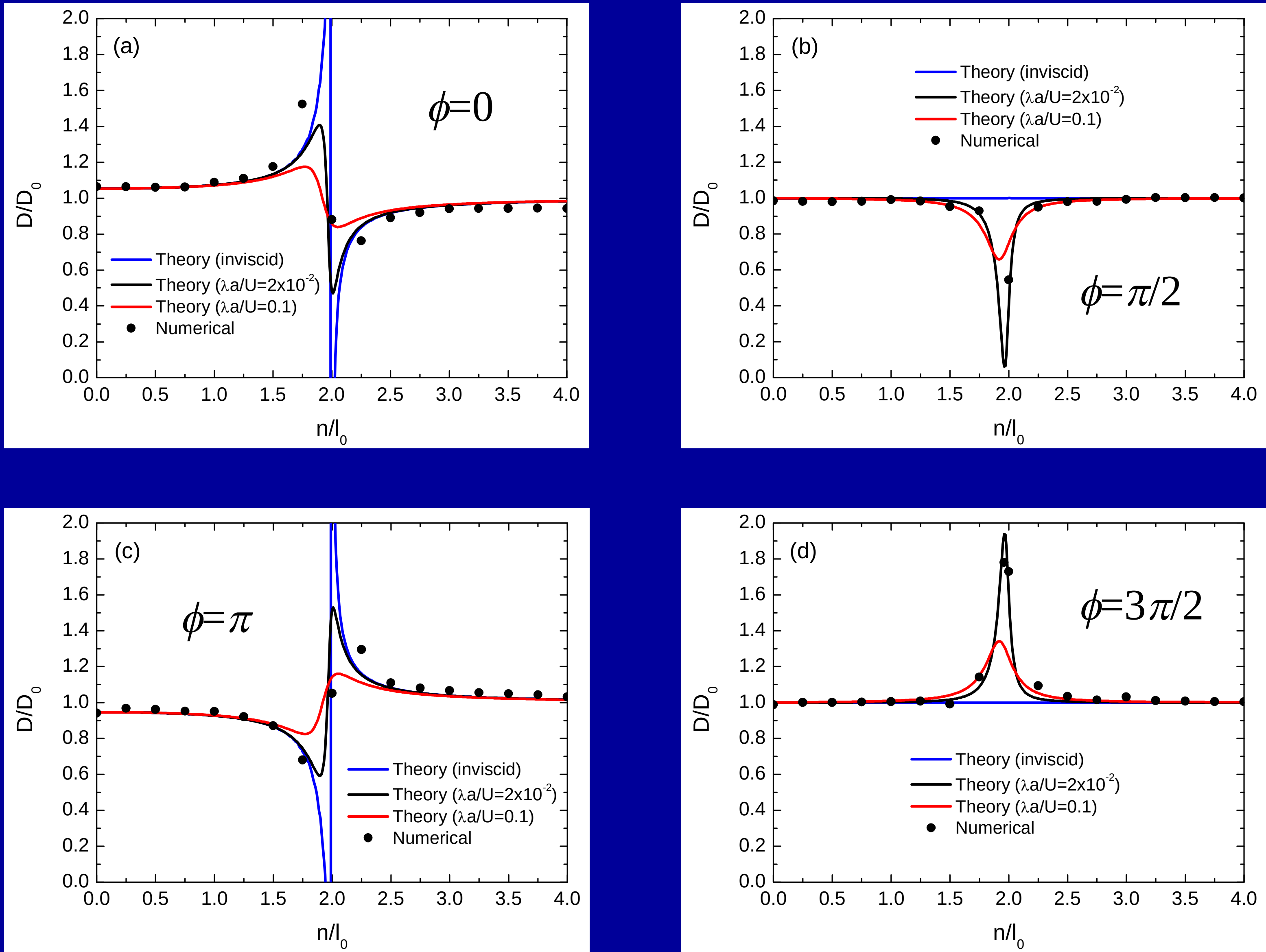
Use 2D nonlinear nonhydrostaitc model called FLEX (Argaín et al. 2009, *BLM*)

- Finite difference method, with implicit time integration
- Terrain-following grid with 192 x 525 grid points
- Domain of **240 km** in horizontal and **44 km** in vertical
- Local grid refinement near surface with Δz **~ 32 m**
- Sponges with Rayleigh damping at lateral and top boundaries
- Raymond & Kuo boundary condition at lateral boundaries
- 4th-order spatial filter applied every 3 time steps
- $\Delta t = 2$ **s**, integration time up to 69 h, typically 17 h.

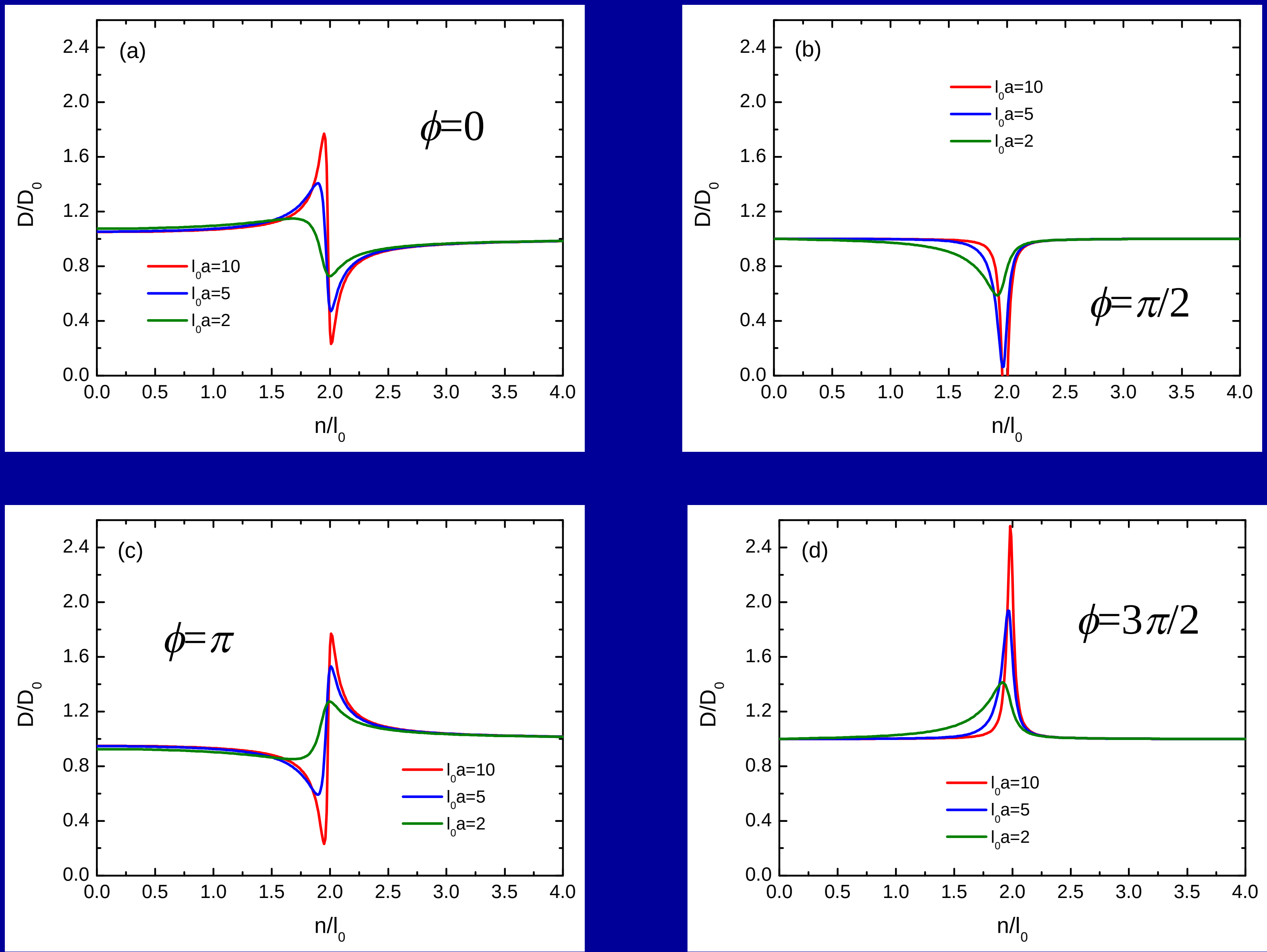


- Anelastic approximation used
- Inviscid flow (no turbulence closure)
- Free-slip surface boundary condition
- U=20 m/s, N=0.01 s^{-1}, a=4,10,20 km, h_0=10 m
- $l_0 a$=2,5,10, $l_0 h_0$=5x10^{-3} (strongly linear flow)

Results - Effect of friction $l_0 a = 5$ $\varepsilon = 0.1$



Non-hydrostatic effects (analytical) $\varepsilon = 0.1$ $\lambda a / U = 2\times10^{-2}$



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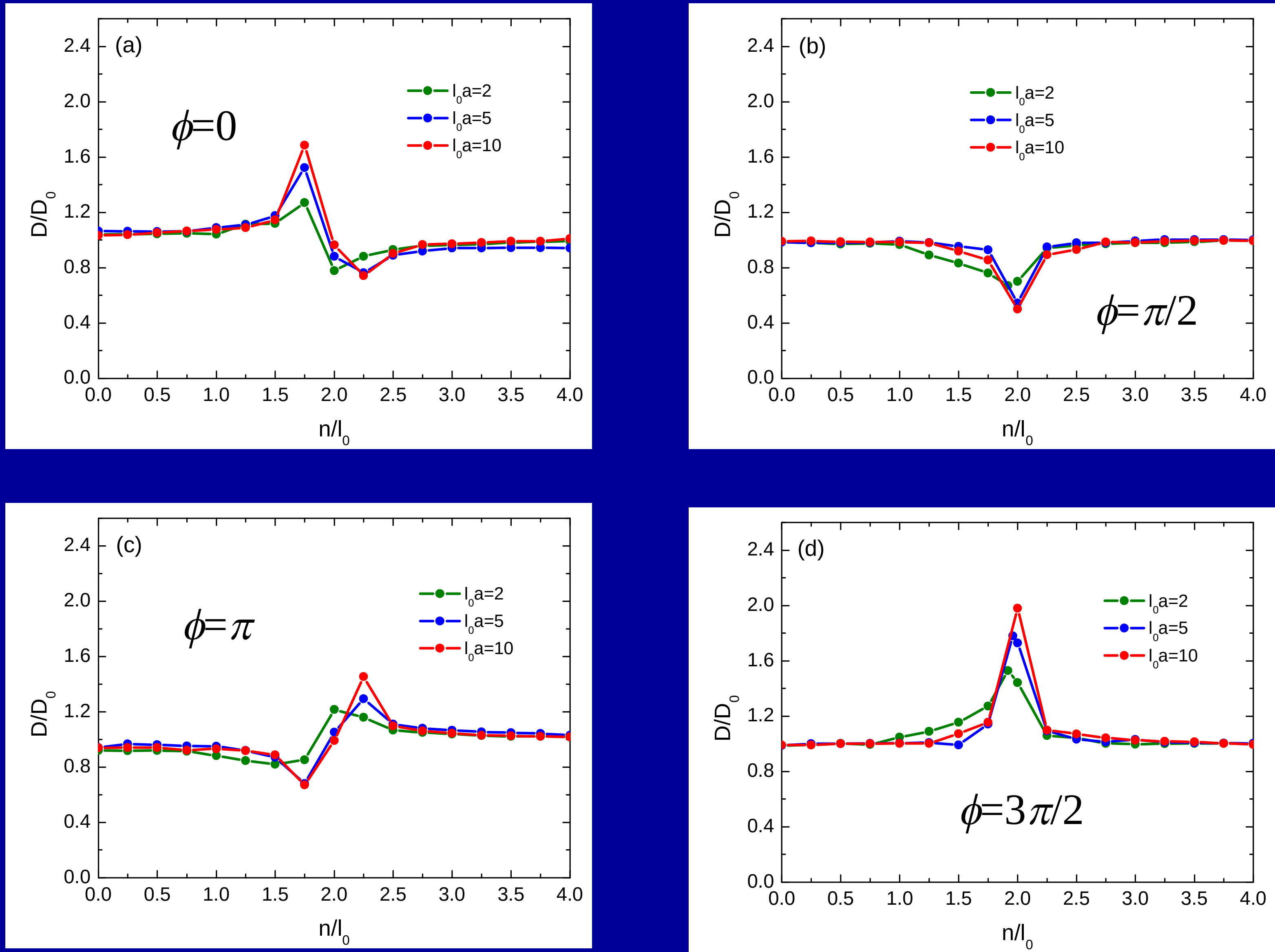
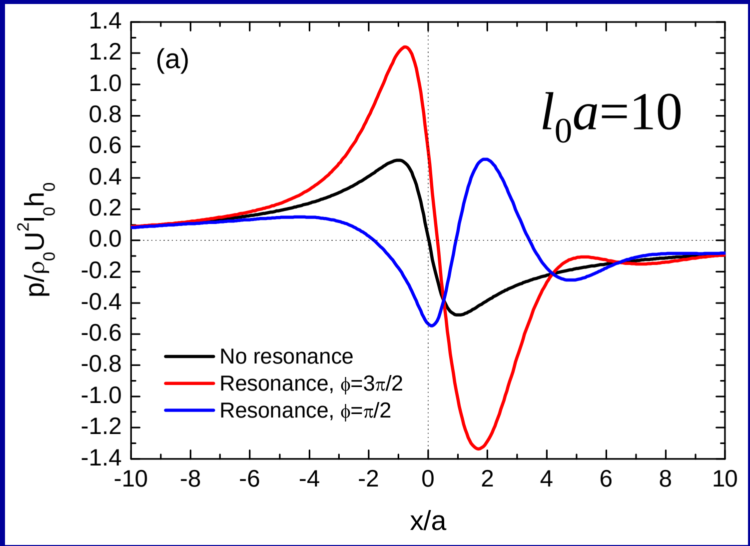
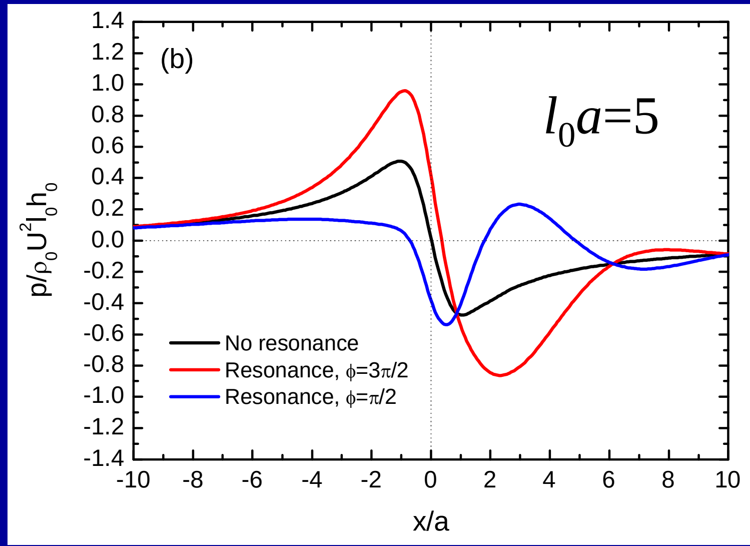
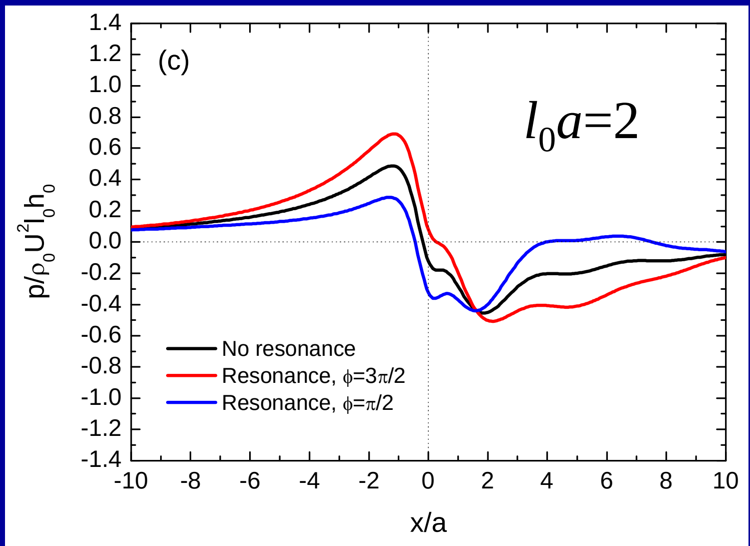


Pressure perturbation (analytical) $\varepsilon = 0.1$ $n/l_0 = 2$

Normalized pressure perturbation, $p/[\rho_0 U^2 l_0 h_0]$ $\lambda a/U = 2\times10^{-2}$





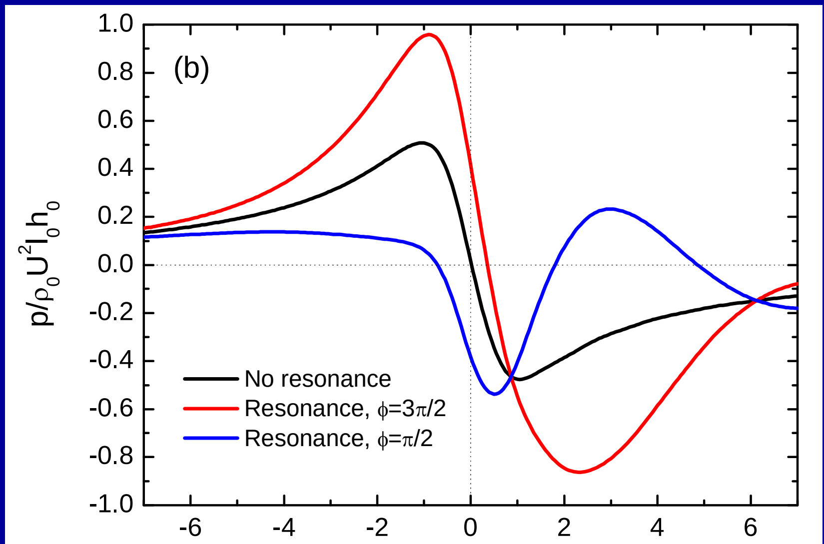


- Pressure perturbation more symmetric (low drag) when $\phi=\pi/2$, and more antisymmetric (high drag) when $\phi=3\pi/2$
- Effect becomes weaker as l_0a decreases

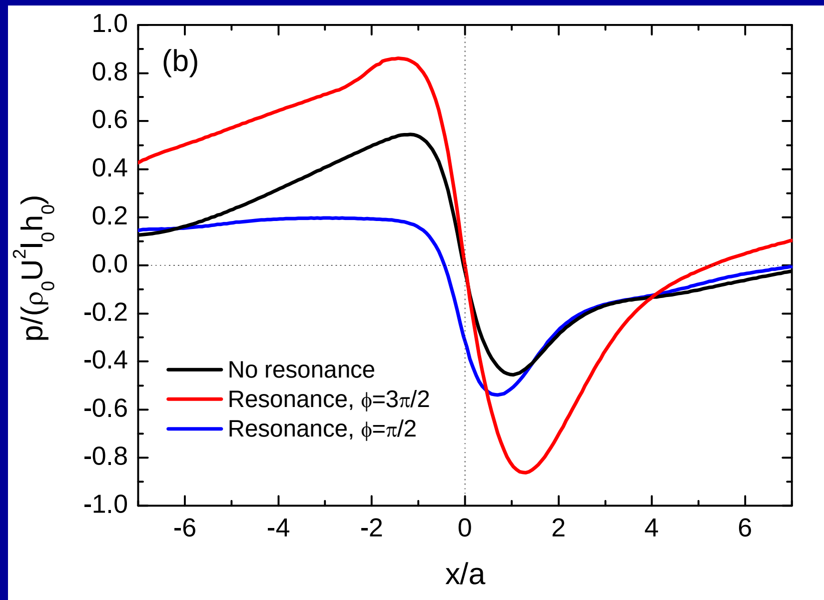
Pressure - comparison with numerical results

Normalized pressure perturbation, $p/[\rho_0 U^2 l_0 h_0]$

$\varepsilon = 0.1$ $n / l_0 = 2$ $l_0 a = 5$



$\lambda a / U = 2 \times 10^{-2}$

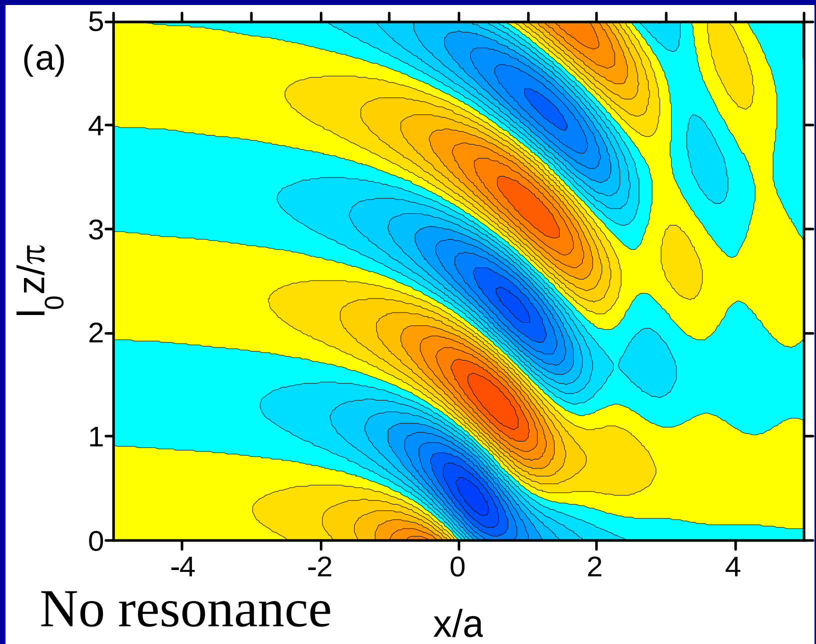


- Relative magnitude of pressure perturbation correctly predicted
- Some details of the flow different – e.g. absence of downstream maximum in blue curve

Flow structure

Normalized vertical velocity perturbation field, $w/[(h_0/a)U]$

Non-resonant case: $\varepsilon = 0$ $l_0 a = 5$



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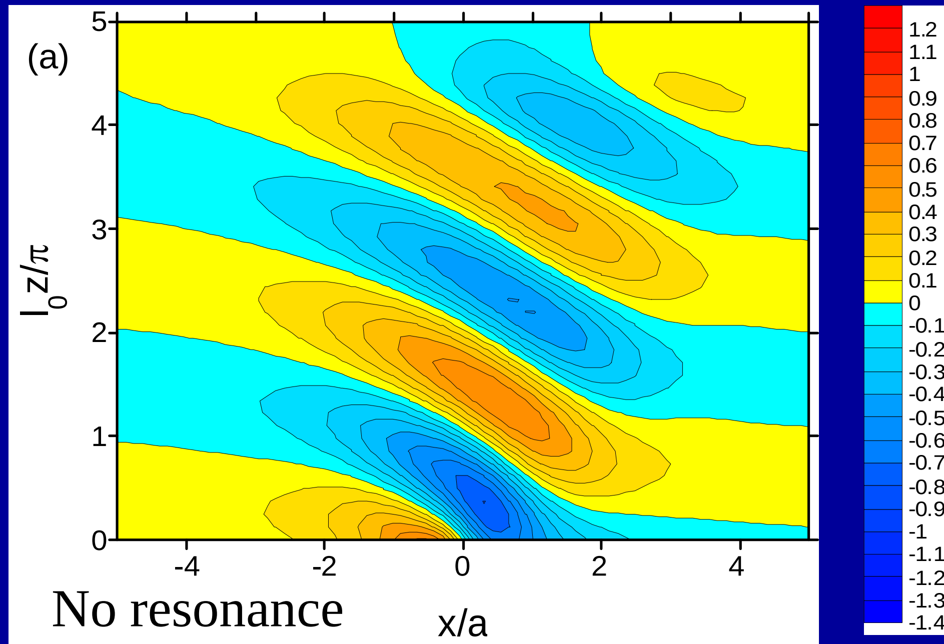
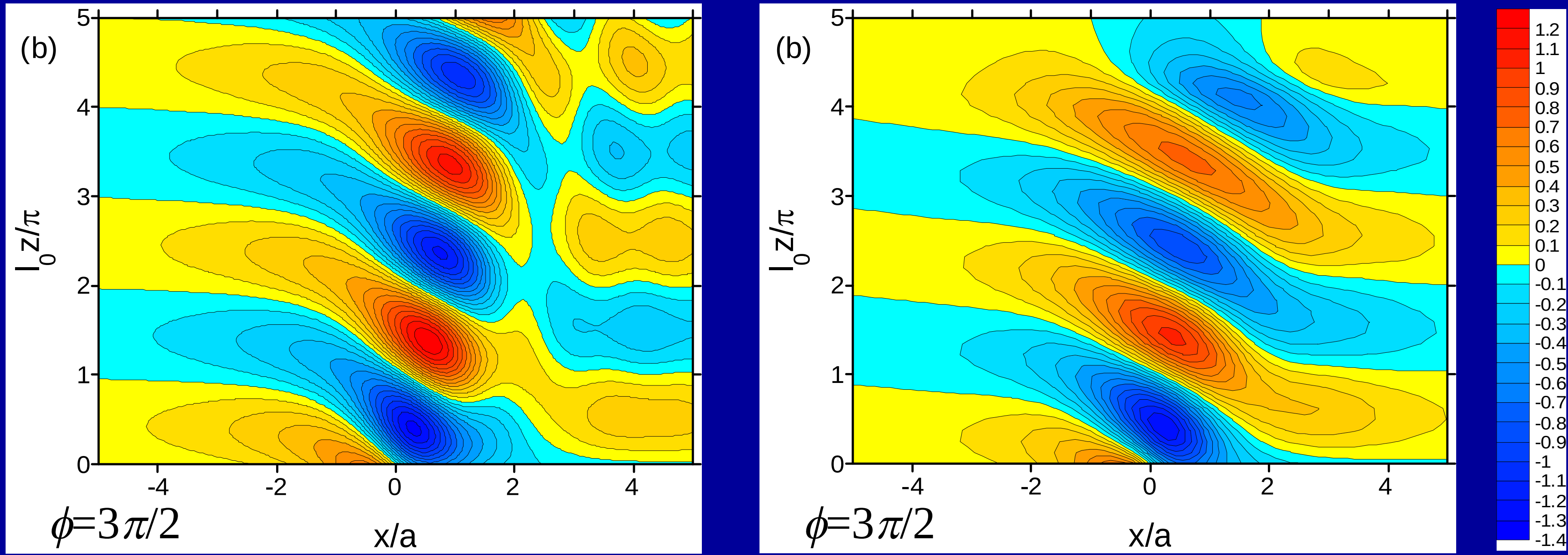


Absorbing layer above $l_0 z/\pi = 4$

Flow structure

Normalized vertical velocity perturbation field, $w/[(h_0/a)U]$

High-drag state: $\varepsilon = 0.1$ $n/l_0 = 2$ $l_0 a = 5$



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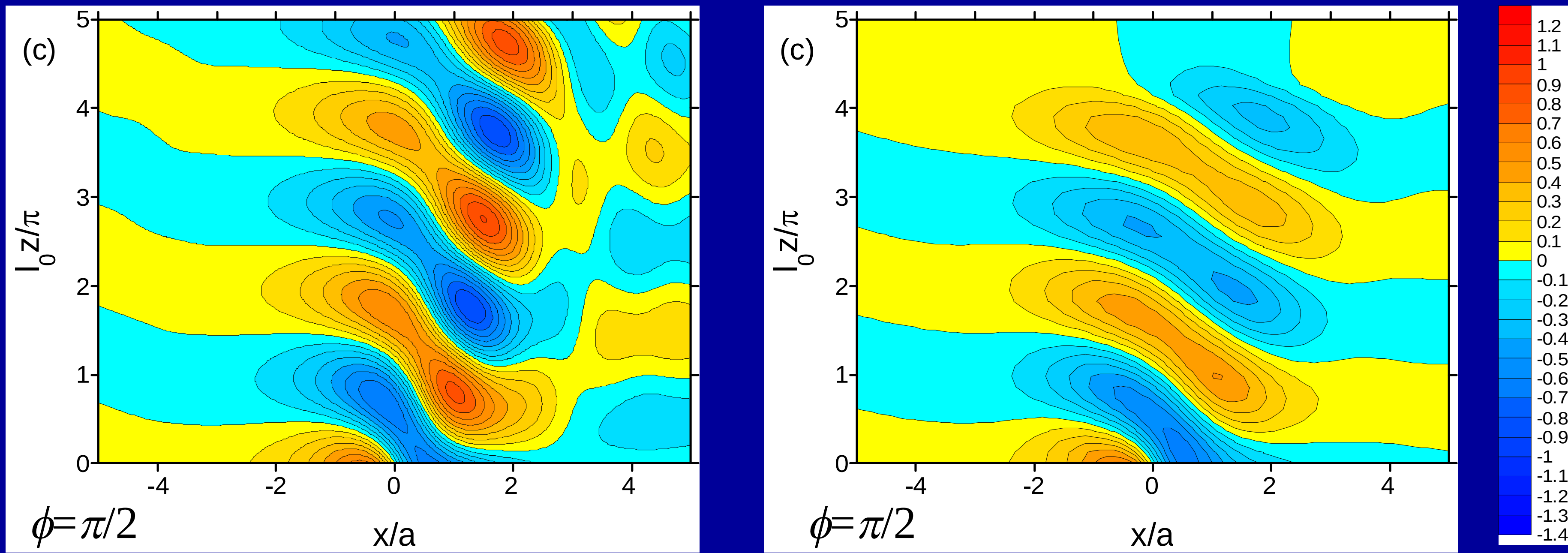
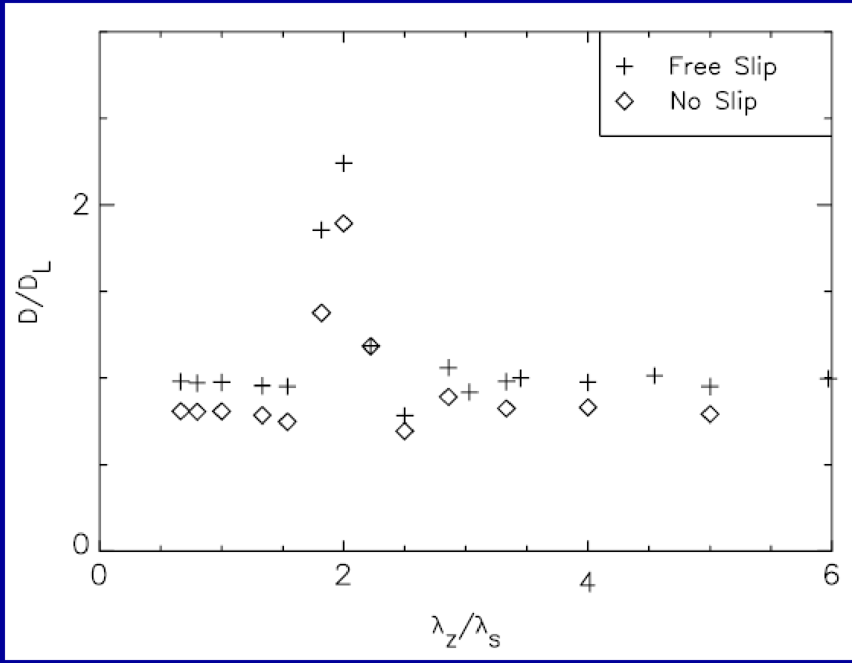


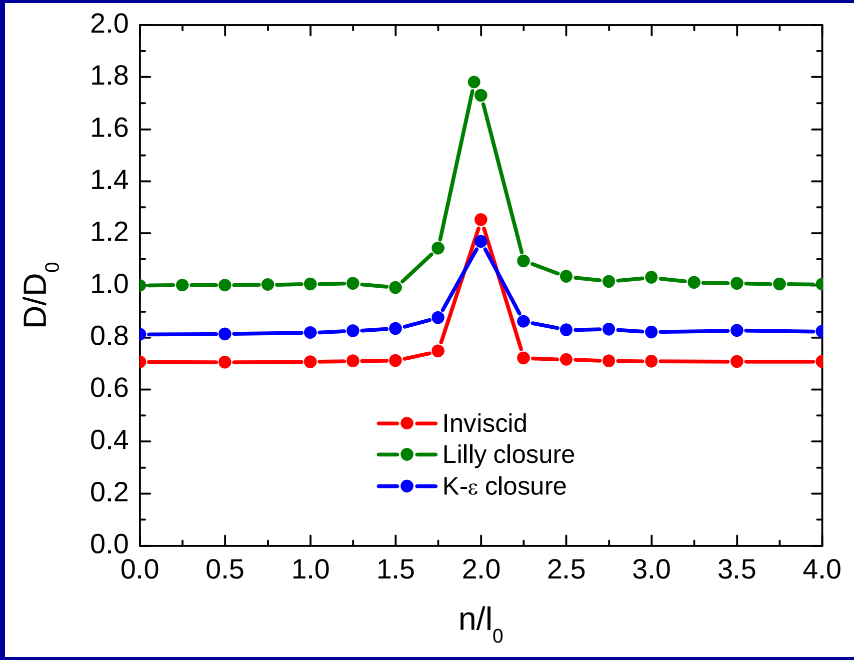
$\lambda a / U = 2\times 10^{-2}$

Absorbing layer above $l_0 z/\pi=4$

Effect of turbulence closures

$\varepsilon = 0.1$ $l_0 a = 5$ $\phi = 3\pi / 2$





(from Wells and Vosper 2010)

- Smagorinsky-type or K-ε turbulence closure
- M-O scaling in surface layer

- Large variation in drag behaviour depending on turbulence closure (and on numerical details in “inviscid” conditions)

Summary

- Mountain wave drag may be significantly enhanced when Scorer parameter oscillates with height
- Gravity wave drag behaviour is reproduced qualitatively in linear framework including nonhydrostatic effects and friction
- Substantial fractional drag ehancement results from resonance when $n/l_0 \approx 2$, even if $\varepsilon = 0.1$ - parametric resonance
- Friction has important impact on drag behaviour, generally moderating resonance
- However, when $\phi = \pi/2$ or $\phi = 3\pi/2$, drag maxima or minima are totally suppressed in inviscid conditions
- Important for numerical models, because both turbulence closures and numerical dissipation introduce ‘friction’
- Non-hydrostatic effects moderate resonance, because of wave dispersion

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